Improving the Quality of Foreign Military Sales Forecasting Using Benford’s Law

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[The views presented herein are solely those of the authors and do not represent the official opinions of DISAM or the Defense Security Cooperation Agency (DSCA). Editor’s note: This article observes that the distribution of historical foreign military sales (FMS) data shows a statistically significant correlation with Benford’s law and therefore one could expect fairly accurate predictions for future FMS if the cautions concerning base data collections are followed. To avoid any unwarranted assumption, understand this is one of several evaluated tools available for predicting future activity.]

Abstract

One approach to forecasting future sales might be called the “bottom-up” approach. In general, one tries to forecast the values of all major customers’ orders for the upcoming year. Then, these are summed to obtain a forecast for the upcoming year’s total sales. This approach can be used in conjunction with other methods, such as examining current sales trends, as part of the overall forecasting process. When using any forecasting method, one needs to understand the quality of the data being used. This paper shows how to use an intriguing mathematical phenomenon called Benford’s Law to measure the quality of the data being used for bottom-up forecasting when large numbers of customer orders are expected.

The Bottom-Up Approach to Forecasting

Good forecasts of future sales often can be built by combining the results of several forecasting approaches. One that can be used might be called the “bottom-up” approach. BusinessDictionary.com (2009) defines a bottom-up sales forecast as a “Method where . . . the sales revenue estimates of each product or product line are combined to compute [the] revenue estimate for the entire firm.” Suppose an organization has regular customers with whom it has done business over the years and those customers provide a large number of sales contracts having a wide range of dollar values. Historically, on any given year, a few new customers might have entered the market; a few customers might have left the market; but, as a general rule, the organization has a regular clientele. When the organization starts its forecasting process for next year’s sales, it might individually meet with its customers to learn what they may desire to buy during that time frame. For example, suppose an interview with Customer A indicates that he or she intends to buy goods and services totaling $1,249,432 on one sales contract, $45,814 on a second sales contract, and $928 on a third sales contract. After the organization meets with each customer and obtains the dollar values for expected sales contracts for the forecast year, the forecaster can then list these sales contracts and their dollar values, add their dollar values, and thereby obtain a forecast for the upcoming year’s sales. The dollar value of each sales contract can be considered a data point.
There are many good further discussions regarding bottom-up forecasting. For example, Kahn discusses key advantages and disadvantages of the approach in his 1998 article “Revisiting Top-Down Versus Bottom-Up Forecasting” [Kahn 1998].

There is a possible disadvantage of bottom-up forecasting in this situation. Some customers will perform due diligence and provide very reasonable sales contract dollar values. However, others might dismiss the request from the organization and provide data points of little or no value. The adage “garbage-in-garbage-out” certainly applies here, and the forecaster needs a way to measure the quality of these data points received to ensure that the resulting sales forecast for the upcoming year is of the best quality possible.

One forecasting data point quality measurement system to use is simple—albeit initially a very unusual approach. First, look again at the list of all sales contracts. Examine just the leading (first) digit of each sales contract. Using the example above, we expand the list by adding a third column as shown below; and we would continue the list for all sales contracts.

<table>
<thead>
<tr>
<th>Sales Contract</th>
<th>Expected Sales</th>
<th>Leading Digit of Expected Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contract 1</td>
<td>$1,249,432</td>
<td>1</td>
</tr>
<tr>
<td>Contract 2</td>
<td>$45,814</td>
<td>4</td>
</tr>
<tr>
<td>Contract 3</td>
<td>$928</td>
<td>9</td>
</tr>
</tbody>
</table>

When the list is complete, determine the percentage of times each leading digit occurs in the list. For example, what percentage of the time does the leading digit “1” occur? If the percentage of occurrence does not coincide with Benford’s Law for a certain leading digit, then this is an indicator of defective input quality; the forecaster should look again at all of the data inputs having that leading digit and perhaps re-contact certain associated customers. The question is, what would you expect each percentage to be?

**Benford’s Law**

Benford’s Law is named for the late Dr. Frank Benford, a physicist formerly at the General Electric Company. In 1938, Dr. Benford noticed that pages of logarithms corresponding to numbers starting with the numeral 1 were much dirtier and more worn than other pages (Browne 1998). In fact, in numerous lists of numbers he then studied from many real-life sources of data, the leading digit “1” occurred more often than the others, namely about 30 percent of the time.

A lively account of Benford’s Law is found in a blog entitled “Fabulous Adventures in Coding.”

While I was poking through my old numeric analysis textbooks to refresh my memory for this series on floating point arithmetic, I came across one of my favorite weird facts about math. A nonzero base-ten integer starts with some digit other than zero. You might naively expect that given a bunch of “random” numbers, you’d see every digit from 1 to 9 about equally often. You would see as many 2’s as 9’s. You would see each digit as the leading digit about 11 percentage of the time. For example, consider a random integer between 100,000 and 999,999. One ninth begin with 1; one ninth begin with 2, etc. But in real-life datasets, that’s not the case at all. If you
just start grabbing thousands or millions of “random” numbers from newspapers and
magazines and books, you soon see that about 30 percent of the numbers begin with
1; and it falls off rapidly from there. About 18 percent begin with 2, all the way down
to less than 5 percent for 9. This oddity was discovered by Newcomb in 1881 and
then rediscovered by Frank Benford, a physicist, in 1937. As often is the case, the fact
became associated with the second discoverer and is now known as Benford’s Law.
Benford’s Law has lots of practical applications. For instance, people who just make
up numbers wholesale on their tax returns tend to pick “average seeming” numbers;
and to humans, “average seeming” means “starts with a five.” People think, I want
something between $1000 and $10000, let’s say $5624. The Internal Revenue Service
(IRS) routinely scans tax returns to find unusually high percentages of leading 5’s
and examines those more carefully. Benford’s result was carefully studied by many
statisticians and other mathematicians, and we now have a multi-base form of the law.
Given a bunch of numbers in base B, we’d expect to see leading digit n approximately
in (1 + 1/n) / in B of the time. But what could possibly explain Benford’s Law?
(Lippert 2005)

This article answers the question “Why does Benford’s Law work?” (at least in many situations)
and shows that it also applies to sales forecasting. This paper is based on the previous research of the
authors (Tichenor, Davis 2008) (Tichenor, Davis 2009).

Logarithms

The first step to understanding why Benford’s Law works is to refresh our minds about exponents
and logarithms (usually abbreviated as “log”).

We are all familiar with how to express numbers using exponents. For example:

\[
\begin{align*}
10^1 &= 10 \\
10^2 &= 100 \\
10^3 &= 1000
\end{align*}
\]

and so on, where the 1, 2, and 3 are exponents. Any positive number can be expressed as 10 to some
power. For example:

\[
\begin{align*}
10^{.3010} &= 1 \\
10^{.4771} &= 2 \\
10^{.6021} &= 3
\end{align*}
\]

and so on. You can check these with a scientific calculator. We can also reverse-engineer these
equations and say that:

\[
\begin{align*}
\log 10 &= 1 \\
\log 100 &= 2 \\
\log 1000 &= 3
\end{align*}
\]

also:

\[
\begin{align*}
\log 1 &= .3010 \\
\log 2 &= .4771 \\
\log 3 &= .6021
\end{align*}
\]

and so on.
Weber-Fechner Law

Our study of the underlying causes of Benford’s Law includes the research of Ernst Heinrich Weber. Weber (Wikipedia 2005) found a form of the law of diminishing returns relationship in humans between stimulus and response: as stimulus increased, response also increased but at a decreasing rate that is logarithmic. For example, if stimulus increased by a factor of 2 (i.e., 100 percent), then response increased by log 2, or .3010. If stimulus increased by a factor of 3, then response increased by a factor of log 3, or .4771. If stimulus increased from a factor of 2 to a factor of 3, then response increased by log 3 – log 2, or .4771 - .3010, or .1761. (Using this line of reasoning, if a stimulus level “increases” by a factor of 1, then there actually is no change in stimulus level and therefore no response.) This important finding is summarized in the below table, and was verified by Sinn (Sinn 2002).

<table>
<thead>
<tr>
<th>Stimulus Level</th>
<th>Response Level Log</th>
<th>Incremental Response</th>
<th>Percent Incremental</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.3010</td>
<td>0.3010</td>
<td>30.10%</td>
</tr>
<tr>
<td>3</td>
<td>0.4771</td>
<td>0.1761</td>
<td>17.61%</td>
</tr>
<tr>
<td>4</td>
<td>0.6021</td>
<td>0.1249</td>
<td>12.49%</td>
</tr>
<tr>
<td>5</td>
<td>0.6990</td>
<td>0.0969</td>
<td>9.69%</td>
</tr>
<tr>
<td>6</td>
<td>0.7782</td>
<td>0.0792</td>
<td>7.92%</td>
</tr>
<tr>
<td>7</td>
<td>0.8451</td>
<td>0.0669</td>
<td>6.69%</td>
</tr>
<tr>
<td>8</td>
<td>0.9031</td>
<td>0.0580</td>
<td>5.80%</td>
</tr>
<tr>
<td>9</td>
<td>0.9542</td>
<td>0.0512</td>
<td>5.12%</td>
</tr>
<tr>
<td>10</td>
<td>1.000</td>
<td>0.0458</td>
<td>4.58%</td>
</tr>
</tbody>
</table>

It is unlikely that a stimulus level will increase from 1 to exactly 2. It could increase to any of numerous intermediate levels, such as 1.04, 1.3, or 1.72. What is important is that 30.10% (.3010) of the possible stimulus levels will range from 1 to 2. Put another way, those leading digits will be a 1 (such as the 1.04, 1.3, or 1.72). In the same way, stimulus levels could range from 2 to 3, such as 2.3, 2.47, or 2.989. The number of stimulus levels that would have a leading digit of 2 will be 17.61%, or .1761.

There is a sidebar that we also need to discuss. If the stimulus level increases from 1 to 2, then the response increases by 30.10 percent. Those familiar with logarithm math would conclude that it is also true that if the stimulus increases from 10 to 20, 100 to 200, 1000 to 2000, etc., then the corresponding responses would also have to increase by factors of 30.10 percent. What is important is the following conclusion. According to the Weber-Fechner Law, if we randomly sample a statistically large number of responses, we will find that about 30.10 percent of them will have a stimulus level starting with a leading digit of 1. We will find that about 17.61 percent of the responses will have a
corresponding stimulus level starting with a 2, and so on. We will find that only 4.58 percent will have a leading digit of 9.

**How Benford’s Law Applies to Bottom-Up Sales Forecasting**

Suppose that sales are a human stimulus and response activity. The response is customer satisfaction, and the stimulus level is measured by the dollar value of the customer’s sales contracts. If we sample a statistically large number of these sales contacts, then we should find that about 30.10 percent of them have a leading digit of 1, about 17.71 percent have a leading digit of 2, and so on through the leading digit of 9—which should occur about 4.58 percent of the time. Benford’s Law would apply.

To test this, we looked at all of the sales contracts used by DSCA over a recent multi-year period. Below is a graph of the results.

![Graph of All Sales Contract First Digit Values Compared to Benford's Law]

The leading digit percentages of sales contracts have statistically the same distribution predicted by Benford’s Law and implied by the Weber-Fechner Law.

**Conclusion**

The agreement of the sales contract data with Benford’s Law is almost identical and is statistically significant. We therefore conclude that selling for DSCA is largely a stimulus and response activity and that it is modeled almost perfectly using Benford’s Law. We also conclude that if bottom-up sales forecasting is done well, then the forecast values of the sales contracts would be close to the eventual actual values and the forecast data points would be distributed according to Benford’s Law. Deviations from Benford’s Law might signify the need to revisit certain individual customer data points.
References


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